Magnetic moment of Δ^{++} baryon in QCD string approach

B.O. Kerbikov
State Research Center
Institute of Theoretical and Experimental Physics,
Moscow, Russia

Abstract

Magnetic moments (m.m.) of the Δ^{++} baryons is computed within the new approach based on the QCD string Hamiltonian. The string tension σ is only dimensionful quantity forming m.m. of both nucleon and Δ^{++}, but color Coulomb and spin-spin interactions cancel each other in nucleon m.m. while in Δ^{++} they add coherently. The result $\mu_{\Delta^{++}} = 4.36\mu_N$ is in good agreement with experimental data.

Recently in [1] a new approach to evaluation of the baryon m.m. has been proposed. It is based on the Feynman-Schwinger (world-line) representation of the 3q Green's function and yields remarkably simple expressions for the m.m. through the only fundamental parameter –string tension σ (and strange quark current mass for strange baryons.) In [1] calculations have been performed for the octet baryons and for Ω^-. The present letter aims at the calculation of the Δ^{++} m.m. and some amendments to our previous treatment of Ω^-.

The case of Δ^{++} is a particular one both from experimental and theoretical sides. Experimentally m.m. of Δ^{++} was measured rather recently and is still subject to substantial uncertainties [2]. On the theoretical side the calculation of the Δ^{++} m.m. causes serious difficulties in various approaches and calls for introduction of several additional parameters (see e.g. recent paper [3] for a brief review of the current status of the sum rules approach to the problem) As will be seen in what follows the treatment of the Δ^{++} in our approach requires taking into account hyperfine and color Coulomb

interactions. This was not the case for octet baryons considered in [1] since for them Coulomb and spin-spin terms extinguished each other [1]. For Ω^- baryon also considered in [1] the situation is the following. As for other baryons the m.m. of Ω^- is predominantly controlled by string tension σ, next comes the contribution from the strange quark current mass m_s, and then Coulomb and hyperfine terms contribute. In [1] the first two factors (σ and m_s) were considered while the present work deals in addition with the two remaining terms listed above. This does not alter the value of the Ω^- m.m. obtained in [1] but allows to take somewhat smaller value of m_s than that used in [1].

We start with a recapitulation of the very few key points of the approach to baryon m.m. developed in [1]. For the $3q$ Green's function one can write the following Feynman–Schwinger (world-line representation [4]-[6])

$$G^{(3q)}(X,Y) = \int \prod_{i=1}^{3} ds_i D z_\mu^{(i)} e^{-K} \langle W_3(X,Y)\rangle, \tag{1}$$

where $X;Y = x^{(i)}; y^{(i)}, i = 1,2,3$, and the integration is along the path $z_\mu^{(i)}(s_i)$ of the i-th quark with s_i playing the role of the proper "time" parameter along the path, and

$$K = \sum_{i=1}^{3} \left(m_i^2 s_i + \frac{1}{4} \int_0^{s_i} \left(\frac{dz_\mu^{(i)}}{d\tau_i} \right) d\tau_i \right). \tag{2}$$

Here m_i is the current quark mass and the three-lobed Wilson loop is a product of the three parallel transporters [4]-[6]. The standard approximation of the QCD string approach is the minimal area law for $\langle W_3 \rangle$

$$\langle W_3 \rangle = \exp(-\sigma \sum_{i=1}^{3} S_i), \tag{3}$$

where S_i is the minimal area of one loop. The next step is to calculate the quark constituent mass ν_i in terms of its current mass m_i and string tension σ. To this end one connects the proper and real times via

$$ds_i = \frac{dt}{2\nu_i(t)}, \tag{4}$$

where $t = z_4^{(i)}(s_i)$ is a common c.m. time on the hypersurface $t = const$ [4, 5]. The new entity, $\nu_i(t)$, being determined from the condition of the minimum

of the corresponding Hamiltonian (see below) plays the role of the quark constituent mass (see [1, 4] for details). Referring to [1, 4] for the derivation of the Hamiltonian from the above defined Green's function $G^{(3q)}$ we write down the final expression containing ν_i as parameters

$$H = \sum_{k=1}^{3}\left(\frac{m_k^2}{2\nu_k} + \frac{\nu_k}{2}\right) + \frac{1}{2m}\left(-\frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \eta^2}\right) + \sigma \sum_{k=1}^{3} |\mathbf{r}^{(k)}|, \tag{5}$$

where $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ are three-body Jacoby coordinates defined as in [6, 7], m is an arbitrary mass parameter which ensures correct dimensions and drops out of final expressions, and $|\mathbf{r}^{(k)}|$ is the distance from the k-sh quark to the string-junction positions which was for simplicity taken coinciding with the c.m. point. It is at this point worth stressing that the QCD string model outlined above is a fully relativistic string model for light current masses, and the "nonrelativistic" appearance of the Hamiltonian (5) is a consequence of the rigorous einbein formalism [8].

Tooled with the Hamiltonian (5) one can use the standard hyperspherical formalism [7], introduce hyperradius $\rho^2 = \xi^2 + \eta^2$, and write the following eigenwalue equation (quarks with equal masses are considered)

$$\frac{d^2\chi}{d\rho^2} + 2\nu\{E_n - W(\rho)\}\chi(\rho) = 0, \tag{6}$$

$$W(\rho) = b\rho + \frac{d}{2\nu\rho^2}, b = \sigma\sqrt{\frac{2}{3}\frac{32}{5\pi}}, d = 15/4. \tag{7}$$

The baryon mass $M_n(\nu)$ is equal to

$$M_n(\nu) = \frac{3m^2}{2\nu} + \frac{3}{2}\nu + E_n(\nu). \tag{8}$$

According to the QCD string prescription [4]-[6] the value of ν is determined as a stationary point of $M_n(\nu)$:

$$\frac{\partial M_n}{\partial \nu} = 0. \tag{9}$$

In passing from (4) to (9) we have changed from $\nu(t)$ depending on the trajectory to the operator ν and finally to the constant ν to be found from the minimum condition (9) – see [4]-[6] for details.

The perturbative gluon exchanges and spin–dependent terms can be self-consistently included into the above picture. Including the Coulomb term and passing to dimensionless quantities x, ε_n and λ defined as

$$x = (2\nu b)^{1/3}\rho, \quad \varepsilon_n = \frac{2\nu E_n}{(2\nu b)^{2/3}}, \quad \lambda = \alpha_s = \frac{8}{3}\left(\frac{10\sqrt{3}\nu^2}{\pi^2\sigma}\right)^{1/3} \equiv \tilde{\lambda}\left(\frac{\nu^2}{\sigma}\right)^{1/3}, \tag{10}$$

where α_s is the strong coupling constant, one arrives at the following reduced equation

$$\left\{-\frac{d^2}{dx^2} + x + \frac{d}{x^2} - \frac{\lambda}{x} - \varepsilon_n(\lambda)\right\} = \chi(x) = 0. \tag{11}$$

Then (9) yields the equation defining the quark dynamical mass ν

$$\varepsilon_n(\lambda)\left(\frac{\sigma}{\nu^2}\right)^{2/3}\left\{1 + \frac{2\lambda}{\varepsilon_n(\lambda)}\left|\frac{d\varepsilon_n}{d\lambda}\right|\right\} + \frac{9}{16}\left(\frac{75\pi^2}{2}\right)^{1/3}\left(\frac{m^2}{\nu^2} - 1\right) = 0. \tag{12}$$

At this point the essential difference of Δ^{++} from p or n arises. It concerns the interplay of the Coulomb and spin-spin interaction (the later is not yet included into (11) and (12)). The spin-spin interaction in baryon made of equal mass quarks results in the shift of E_n equal to

$$\delta E_n = \frac{16}{9}\frac{\alpha_s}{\nu^2}\sum_{i>j}\mathbf{s}_i\mathbf{s}_j\delta(\mathbf{r}_{ij}). \tag{13}$$

For Δ^{++} summation over (i,j) yields a factor 3/4 corresponding to positive interaction energy [9], i.e. in Δ^{++} spin-spin interaction acts coherently with the Coulomb one. In nucleon the corresponding factor is –4/3 resulting in reverse situation. This distintion realizes in the fact that the masses of quarks forming the nucleon remain "unrenormalized" due to Coulomb and spin-spin interaction [1] while the masses of quarks forming Δ^{++} substantially increase as shown below.

To include the term (13) into equation (12) one has to smear the delta functions over small regions. This procedure has been done by two independent methods in [10] with the result $\langle\delta(\mathbf{r}_{ij})\rangle = \delta_0\nu^{3/2}, \delta_0 = 2.64\cdot 10^{-2}$ GeV$^{3/2}$. With spin-spin interaction included Eq. (12) for Δ^{++} takes the form

$$\nu^2 - \frac{16}{9}\left(\frac{2}{75\pi^2}\right)^{1/3}\varepsilon(0)\sigma^{2/3}\nu^{2/3}\left[1 + \frac{\tilde{\lambda}}{\varepsilon(0)}\left|\frac{d\varepsilon}{d\lambda}\right|_{\lambda=0}\left(\frac{\nu^2}{\sigma}\right)^{1/3}\right] - \frac{4\pi}{9}\alpha_s\delta_0\nu^{1/3} = 0. \tag{14}$$

Only terms linear in Coulomb coupling constant $\tilde{\lambda}$ are kept in (14), the corresponding expansion parameter is $\tilde{\lambda}/\varepsilon(0) \simeq 1/4$. Eq(14) has been solved for the following set of parameters

$$\sigma = 0.15 \mathrm{GeV}^2, \quad \alpha_s = 0.39. \tag{15}$$

The string tension value (15) which is smaller than in meson case is in line with baryon calculations by Capstick and Isgur [11]. A similar smaller value of σ is implied by recent lattice calculations by Bali [12]. Solving (14) with the above set of parameters one gets

$$\nu_\Delta = 0.43 \mathrm{GeV}, \tag{16}$$

which should be compared to $\nu_N = c\sqrt{\sigma} = 0.37$ GeV, where $C = 0.957$ is a constant calculated in [1].

Now we turn directly to Δ^{++} magnetic moment. The magnetic momet interaction term is included into $G^{(3q)}$ in a straightforward way [1, 4]-[6] resulting in

$$\mu_{\Delta^{++}} = 3 \left\langle \psi_{\Delta^{++}} \left| \frac{e_3 \sigma_z^{(3)}}{2\nu_{\Delta^{++}}} \right| \psi_{\Delta^{++}} \right\rangle, \tag{17}$$

where $e_3 = e_u = 2/3$. The structure of this matrix element with $\nu_{\Delta^{++}}$ in the denomitor may be considered as an additional evidence that the quantity ν first introduced by (4) has the physical meaning of the quark constituent mass.

The calculation of the matrix element (17) is trivial provident one considers only totally symmetric coordinate wave functions, i.e. the lowest hyperspherical harmonic. It is known that the contribution of higher harmonics into normalization does not exceed few percent [7, 13]. Then (17) yields

$$\mu_{\Delta^{++}} = \frac{2m_p}{\nu_\Delta} \simeq 4.36 \mu_N, \tag{18}$$

which is in good agreement with the experimental value $\mu_{\Delta^{++}} = (4.52 \pm 0.50 \pm 0.45)\mu_N$ [2].

The treatment of the Δ^{++} baryon presented above makes it possible to reexamine the case of Ω^- baryon considered in [1] and to improve upon the value of the strange quark current mass used in [1]. Namely in [1] the constituent mass ν_Ω was calculated using Eq.(12) above and then m.m. of Ω^- was obtained as a matrix element similar to (17). Coulomb and spin-spin

terms in Ω^- were neglected since their contribution is smaller than that of the strange quark constituent mass m_s. If following the lines outlined above these hitherto omitted terms are included into the treatment of the Ω^- m.m. all the values of the baryon m.m. presented in [1] (see also Table 1 below) remain unchanged but they are reproduced with smaller values of m_s, namely with $m_s = 0.15$ GeV instead of $m'_s = 0.245$ GeV used in [1]. The new value of m_s agrees with the result $m_s = 0.175 \pm 0.025$ GeV deduced by Leutwyler [14].

In Table 1 we summarize the results on baryon m.m. obtained in the present work and in [1]. The typical deviations from the experimental values are about 10% which is remarkably successful keeping in mind plentiful possible corrections (meson exchanges, higher harmonics, etc).

The author is especially indebted to Yu.A.Simonov for formulating the problem, discussions and remarks. The author is grateful to Yu.S.Kalashnikova for numerous enlighting discussions and suggestions and to A.M.Badalian and N.O.Agasian for useful remarks.

The financial support of the grant RFFI 00-02-17836 and RFFI 96-15-96740 are gratefully acknowledged.

Table 1. Magnetic moments of baryons (in nuclear magnetons) in comparison with experimental data from PDG []

Baryon	p	n	Δ^{++}	Λ	Σ^-	Σ^0	Σ^+	Ξ^-	Ξ^0	Ω^-
Ref.[1] and present work	2.54	-1.69	4.36	-0.69	-0.90	0.80	2.48	-0.63	-1.49	-2.04
Experiment	2.79	-1.91	4.52	-0.61	-1.16		2.46	-0.65	-1.25	-2.02

References

[1] B.O.Kerbikov and Yu.A.Simonov, Baryon magnetic moments in the QCD string approach, hep-ph/0001243.

[2] PDG group, Review of particle Physics, The European Physical Journal **C3** (1998)

[3] Y.M.Aliev, A.Özpineci, M.Savci, Magnetic moments of Δ baryons in light cone QCD sum rules, hep-ph/0002228.

[4] Yu.A.Simonov, Nucl. Phys. **B307** (1988) 512; Phys. Lett. **B226** (1989) 151; hep-ph/9911237.

[5] A.Yu.Dubin, A.B.Kaidalov and Yu.A.Simonov, Phys. Lett. **B323** (1994) 41

[6] M.Fabre de la Ripelle and Yu.A.Simonov, Ann. Phys. (N.Y.) **212** (1991) 235

[7] Yu.A.Simonov, Yad. Fiz. **3** (1966) 630;
A.M.Badalian, Yu.A.Simonov, Yad. Fiz. **3** (1966) 1032;
F.Calogero, Yu. A.Simonov Phys. Rev. **183** (1968) 869.

[8] L.Brink, P.Di Vecchia, P.Howe, Nucl. Phys. **B118** (1977) 76;
A.M.Polyakov, Gauge fields and strings, Harwood Academic, 1987;
A.Yu.Dubin, JETP Lett. 56 (1992) 545;
Yu.S.Kalashnikova, A.V,Nefediev, Phys. At. Nucl. **60** (1997) 1389; **61** (1998) 785.

[9] R.L.Jaffe, Color, spin, and flavour-dependent forces in quantum chromodynamics, hep-ph/0001123, January 2000.

[10] B.O.Kerbikov, M.I.Polikarpov and L.V.Shevchenko, Nucl. Phys. **B331** (1990) 19;
A.M.Badalian, Unified description of ground state mesons and baryons in a potential model, Preprint ITEP-21, 1987.

[11] S.Capstick and N.Isgur, Phys. Rev. **D34** (1986) 2809

[12] G.Bali, QCD forces and heavy quark bound states, hep-ph/0001312, January 2000.

[13] Yu.S.Kalashnikova, I.M.Naridetskii, and Yu.A.Simonov, Yad. Fiz. **46** (1987) 1181.

[14] H.Leutwyler, The masses of the light quarks, CERN-TH/96-25